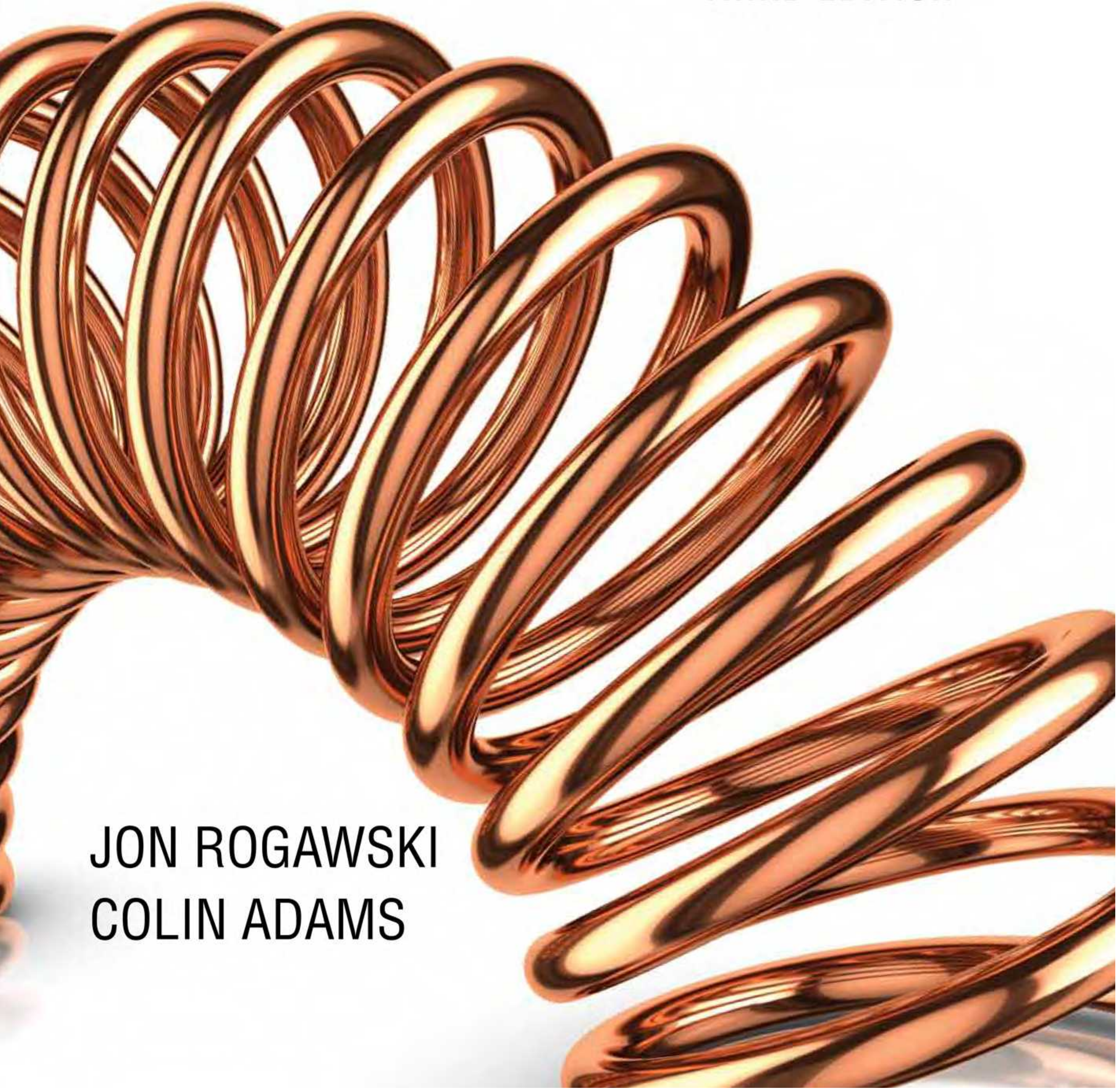
CALCULUS

THIRD EDITION



JON ROGAWSKI
COLIN ADAMS

ALGEBRA

Lines

Slope of the line through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = mx + b$$

Point-slope equation of line through $P_1 = (x_1, y_1)$ with slope m:

$$y - y_1 = m(x - x_1)$$

Point-point equation of line through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$y - y_1 = m(x - x_1) \quad \text{where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Lines of slope m_1 and m_2 are parallel if and only if $m_1 = m_2$.
Lines of slope m_1 and m_2 are perpendicular if and only if $m_1 = -\frac{1}{m_2}$.

Circles

Equation of the circle with center (a, b) and radius r:

$$(x - a)^2 + (y - b)^2 = r^2$$

Distance and Midpoint Formulas

Distance between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\overline{P_1 P_2}$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Laws of Exponents

$x^m x^n = x^{m+n}$ $\qquad \frac{x^m}{x^n} = x^{m-n}$ $\qquad (x^m)^n = x^{mn}$

$x^{-n} = \frac{1}{x^n}$ $\qquad (xy)^n = x^n y^n$ $\qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$x^{1/n} = \sqrt[n]{x}$ $\qquad \sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$ $\qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

$x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

Special Factorizations

$x^2 - y^2 = (x + y)(x - y)$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Binomial Theorem

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

where $\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \cdots \cdot k}$

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$.

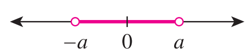
If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

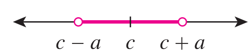
If $a < b$ and $c < 0$, then $ca > cb$.

$|x| = x \quad$ if $x \geq 0$

$|x| = -x \quad$ if $x \leq 0$



$|x| < a$ means $-a < x < a$.



$|x - c| < a$ means $c - a < x < c + a$.

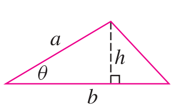
GEOMETRY

Formulas for area A, circumference C, and volume V

Triangle

$A = \frac{1}{2}bh$

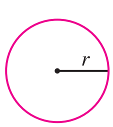
$\;\;= \frac{1}{2}ab\sin\theta$



Circle

$A = \pi r^2$

$C = 2\pi r$

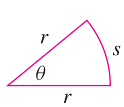


Sector of Circle

$A = \frac{1}{2}r^2\theta$

$s = r\theta$

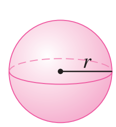
(θ in radians)



Sphere

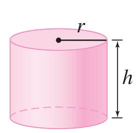
$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$



Cylinder

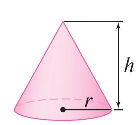
$V = \pi r^2 h$



Cone

$V = \frac{1}{3}\pi r^2 h$

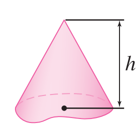
$A = \pi r\sqrt{r^2 + h^2}$



Cone with arbitrary base

$V = \frac{1}{3}Ah$

where A is the area of the base



Pythagorean Theorem: For a right triangle with hypotenuse of length c and legs of lengths a and b, $c^2 = a^2 + b^2$.

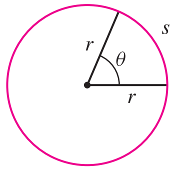
TRIGONOMETRY

Angle Measurement

π radians $= 180°$

$1° = \dfrac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \dfrac{180°}{\pi}$

$s = r\theta \quad (\theta \text{ in radians})$

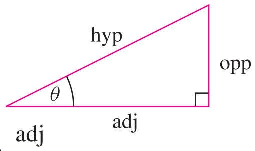


Right Triangle Definitions

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\text{opp}}{\text{adj}} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\text{adj}}{\text{opp}}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{hyp}}{\text{adj}} \qquad \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{hyp}}{\text{opp}}$



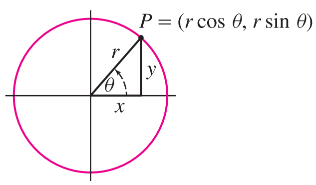
Trigonometric Functions

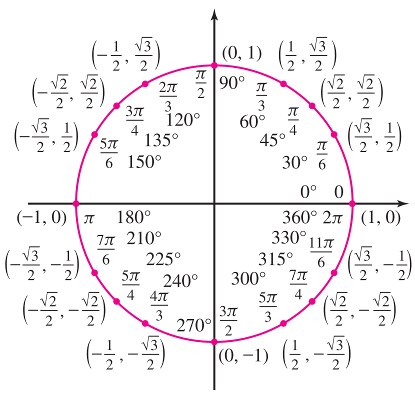
$\sin\theta = \dfrac{y}{r} \qquad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r} \qquad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$

$\displaystyle\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1 \qquad \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = 0$





Fundamental Identities

$\sin^2\theta + \cos^2\theta = 1 \qquad \sin(-\theta) = -\sin\theta$

$1 + \tan^2\theta = \sec^2\theta \qquad \cos(-\theta) = \cos\theta$

$1 + \cot^2\theta = \csc^2\theta \qquad \tan(-\theta) = -\tan\theta$

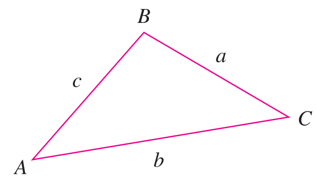
$\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta \qquad \sin(\theta + 2\pi) = \sin\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta \qquad \cos(\theta + 2\pi) = \cos\theta$

$\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta \qquad \tan(\theta + \pi) = \tan\theta$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Addition and Subtraction Formulas

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\sin(x - y) = \sin x\cos y - \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\cos(x - y) = \cos x\cos y + \sin x\sin y$

$\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x\tan y}$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x\tan y}$

Double-Angle Formulas

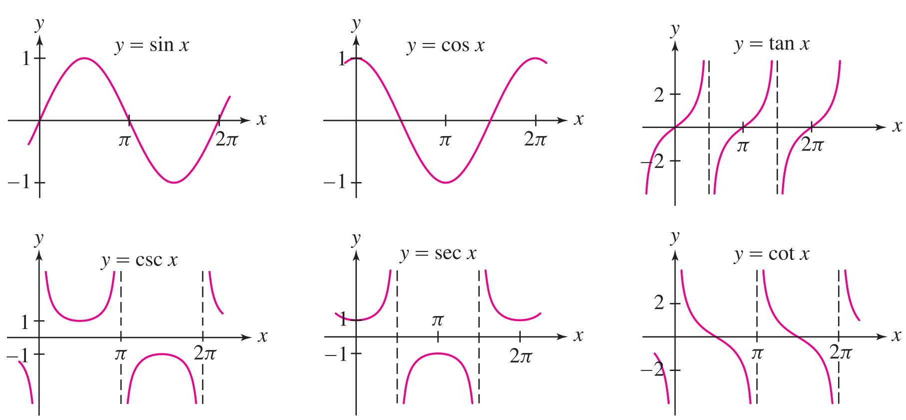
$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\sin^2 x = \dfrac{1 - \cos 2x}{2} \qquad \cos^2 x = \dfrac{1 + \cos 2x}{2}$

Graphs of Trigonometric Functions



THIRD EDITION

CALCULUS

JON ROGAWSKI
University of California, Los Angeles

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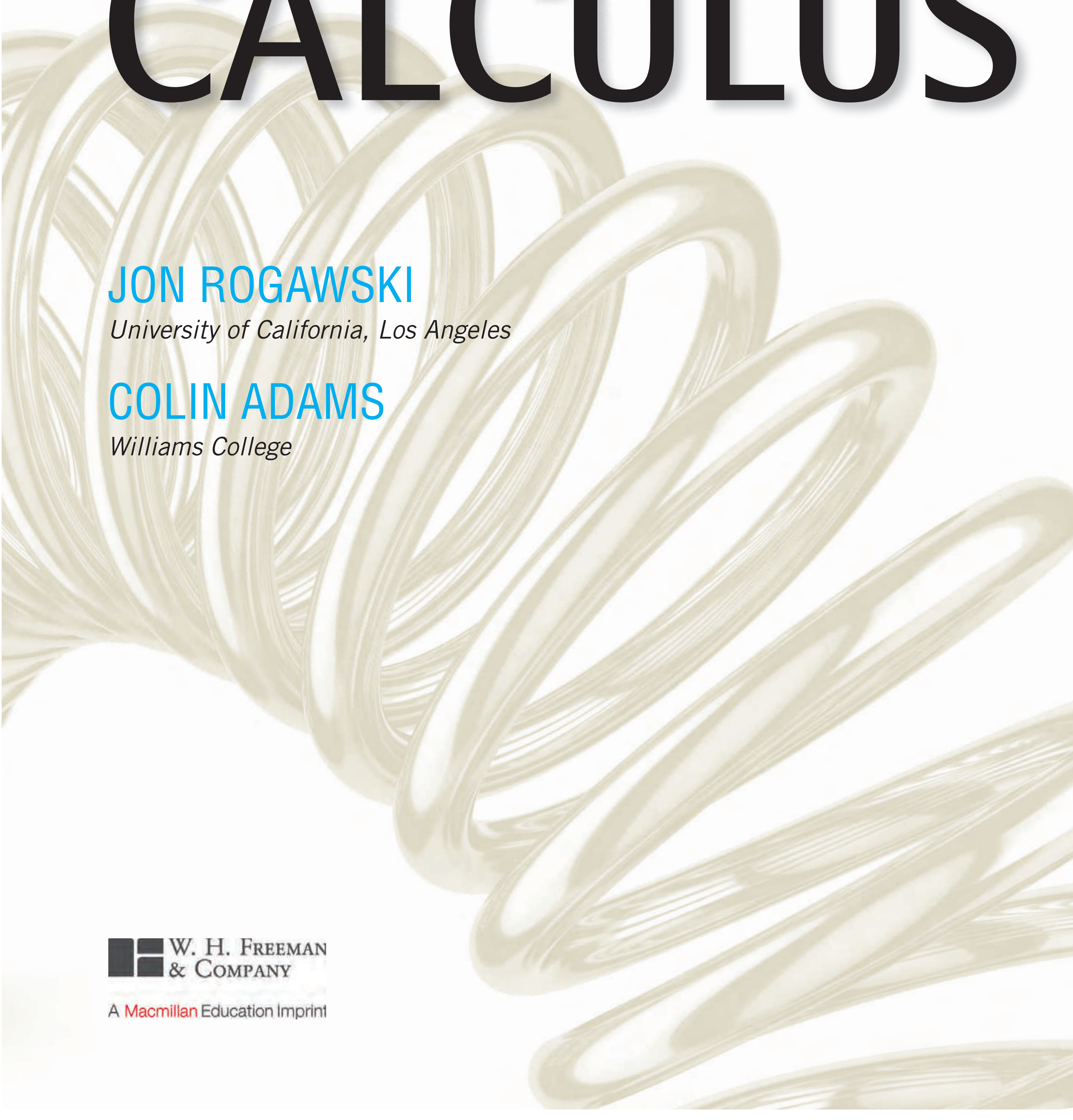

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TO ALEXA AND COLTON –Colin

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COLIN ADAMS

Colin Adams is the Thomas T. Read professor of Mathematics at Williams College, where he has taught since 1985. Colin received his undergraduate degree from MIT and his PhD from the University of Wisconsin. His research is in the area of knot theory and low-dimensional topology. He has held various grants to support his research, and written numerous research articles.

Colin is the author or co-author of *The Knot Book*, *How to Ace Calculus: The Streetwise Guide*, *How to Ace the Rest of Calculus: The Streetwise Guide*, *Riot at the Calc Exam and Other Mathematically Bent Stories*, *Why Knot?*, *Introduction to Topology: Pure and Applied*, and *Zombies & Calculus*. He co-wrote and appears in the videos "The Great Pi vs. E Debate" and "Derivative vs. Integral: the Final Smackdown."

He is a recipient of the Haimo National Distinguished Teaching Award from the Mathematical Association of America (MAA) in 1998, an MAA Polya Lecturer for 1998-2000, a Sigma Xi Distinguished Lecturer for 2000-2002, and the recipient of the Robert Foster Cherry Teaching Award in 2003.

Colin has two children and one slightly crazy dog, who is great at providing the entertainment.

JON ROGAWSKI

As a successful teacher for more than 30 years, Jon Rogawski listened and learned much from his own students. These valuable lessons made an impact on his thinking, his writing, and his shaping of a calculus text.

Jon Rogawski received his undergraduate and master's degrees in mathematics simultaneously from Yale University, and he earned his PhD in mathematics from Princeton University, where he studied under Robert Langlands. Before joining the Department of Mathematics at UCLA in 1986, where he was a full professor, he held teaching and visiting positions at the Institute for Advanced Study, the University of Bonn, and the University of Paris at Jussieu and Orsay.

Jon's areas of interest were number theory, automorphic forms, and harmonic analysis on semisimple groups. He published numerous research articles in leading mathematics journals, including the research monograph *Automorphic Representations of Unitary Groups in Three Variables* (Princeton University Press). He was the recipient of a Sloan Fellowship and an editor of the Pacific Journal of Mathematics and the Transactions of the AMS.

Sadly, Jon Rogawski passed away in September 2011. Jon's commitment to presenting the beauty of calculus and the important role it plays in students' understanding of the wider world is the legacy that lives on in each new edition of *Calculus*.

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Additional content can be accessed online via **LaunchPad**:

ADDITIONAL PROOFS

- L'Hôpital's Rule
- Error Bounds for Numerical Integration
- Comparison Test for Improper Integrals

ADDITIONAL CONTENT

- Second Order Differential Equations
- Complex Numbers

PREFACE

ABOUT *CALCULUS*

On Teaching Mathematics

I consider myself very lucky to have a career as a teacher and practitioner of mathematics. When I was young, I decided I wanted to be a writer. I loved telling stories. But I was also good at math, and, once in college, it didn't take me long to become enamored with it. I loved the fact that success in mathematics does not depend on your presentation skills or your interpersonal relationships. You are either right or you are wrong and there is little subjective evaluation involved. And I loved the satisfaction of coming up with a solution. That intensified when I started solving problems that were open research questions that had previously remained unsolved.

So, I became a professor of mathematics. And I soon realized that teaching mathematics is about telling a story. The goal is to explain to students in an intriguing manner, at the right pace, and in as clear a way as possible, how mathematics works and what it can do for you. I find mathematics immensely beautiful. I want students to feel that way, too.

On Writing a Calculus Text

I had always thought I might write a calculus text. But that is a daunting task. These days, calculus books average over a thousand pages. And I would need to convince myself that I had something to offer that was different enough from what already appears in the existing books. Then, I was approached about writing the third edition of Jon Rogawski's calculus book. Here was a book for which I already had great respect. Jon's vision of what a calculus book should be fit very closely with my own. Jon believed that as math teachers, how we say it is as important as what we say. Although he insisted on rigor at all times, he also wanted a book that was written in plain English, a book that could be read and that would entice students to read further and learn more. Moreover, Jon strived to create a text in which exposition, graphics, and layout would work together to enhance all facets of a student's calculus experience.

In writing his book, Jon paid special attention to certain aspects of the text:

1. Clear, accessible exposition that anticipates and addresses student difficulties.

2. Layout and figures that communicate the flow of ideas.

3. Highlighted features that emphasize concepts and mathematical reasoning: Conceptual Insight, Graphical Insight, Assumptions Matter, Reminder, and Historical Perspective.

4. A rich collection of examples and exercises of graduated difficulty that teach basic skills, problem-solving techniques, reinforce conceptual understanding, and motivate calculus through interesting applications. Each section also contains exercises that develop additional insights and challenge students to further develop their skills.

Coming into the project of creating the third edition, I was somewhat apprehensive. Here was an already excellent book that had attained the goals set for it by its author. First and foremost, I wanted to be sure that I did it no harm. On the other hand, I have been teaching calculus now for 30 years, and in that time, I have come to some conclusions about what does and does not work well for students.

As a mathematician, I want to make sure that the theorems, proofs, arguments and development are correct. There is no place in mathematics for sloppiness of any kind. As a teacher, I want the material to be accessible. The book should not be written at the mathematical level of the instructor. Students should be able to use the book to learn the material, with the help of their instructor. Working from the high standard that Jon set, I have tried hard to maintain the level of quality of the previous edition while making the changes that I believe will bring the book to the next level.

Placement of Taylor Polynomials

Taylor polynomials appear in Chapter 9, before infinite series in Chapter 11. The goal here is to present Taylor polynomials as a natural extension of linear approximation. When teaching infinite series, the primary focus is on convergence, a topic that many students find challenging. By the time we have covered the basic convergence tests and studied the convergence of power series, students are ready to tackle the issues involved in representing a function by its Taylor series. They can then rely on their previous work with Taylor polynomials and the error bound from Chapter 9. However, the section on Taylor polynomials is written so that you can cover this topic together with the materials on infinite series if this order is preferred.

Careful, Precise Development

W. H. Freeman is committed to high quality and precise textbooks and supplements. From this project's inception and throughout its development and production, quality and precision have been given significant priority. We have in place unparalleled procedures to ensure the accuracy of the text:

- Exercises and Examples
- Exposition
- Figures
- Editing
- Composition

Together, these procedures far exceed prior industry standards to safeguard the quality and precision of a calculus textbook.

New to the Third Edition

There are a variety of changes that have been implemented in this edition. Following are some of the most important.

MORE FOCUS ON CONCEPTS The emphasis has been shifted to focus less on the memorization of specific formulas, and more on understanding the underlying concepts. Memorization can never be completely avoided, but it is in no way the crux of calculus. Students will remember how to apply a procedure or technique if they see the logical progression that generates it. And they then understand the underlying concepts rather than seeing the topic as a black box in which you insert numbers. Specific examples include:

- **(Section 1.2)** Removed the general formula for the completion of a square and instead, emphasized the method so students need not memorize the formula.
- **(Section 8.2)** Changed the methods for evaluating trigonometric integrals to focus on techniques to apply rather than formulas to memorize.
- **(Chapter 10)** Discouraged the memorization of solutions of specific types of differential equations and instead, encouraged the use of methods of solution.
- **(Section 13.2)** Decreased number of formulas for parametrizing a line from two to one, as the second can easily be derived from the first.
- **(Section 13.6)** De-emphasized the memorization of the various formulas for quadric surfaces. Instead, moved the focus to slicing with planes to find curves and using those to determine the shape of the surface. These methods will be useful regardless of the type of surface it is.
- **(Section 15.4)** Decreased the number of essential formulas for linear approximation of functions of two variables from four to two, providing the background to derive the others from these.

CHANGES IN NOTATION There are numerous notational changes. Some were made to bring the notation more into line with standard usage in mathematics and other fields in which mathematics is applied. Some were implemented to make it easier for students to remember the meaning of the notation. Some were made to help make the corresponding concepts that are represented more transparent. Specific examples include:

- **(Section 4.5)** Presented a new notation for graphing that gives the signs of the first and second derivative and then simple symbols (slanted up and down arrows and up and down u's) to help the student keep track of when the graph is increasing or decreasing and concave up or concave down over the given interval.
- **(Section 8.1)** Simplified the notation for integration by parts and provided a visual method for remembering it.
- **(Chapter 11)** Changed names of the various tests for convergence/divergence of infinite series to evoke the usage of the test and thereby make it easier for students to remember them.
- **(Chapters 14–18)** Rather than using $c(t)$ for a path, we consistently switched to the vector-valued function $\mathbf{r}(t)$. This also allowed us to replace $d\mathbf{s}$ with $d\mathbf{r}$ as a differential, which means there is less likely to be confusion with $d\mathbf{s}$, dS and $d\mathbf{S}$.

MORE EXPLANATIONS OF DERIVATIONS Occasionally, in the previous edition, a result was given and verified, without motivating where the derivation came from. I believe it is important for students to understand how someone might come up with a particular result, thereby helping them to picture how they might themselves one day be able to derive results.

- **(Section 9.3)** Developed the center of mass formulas by first discussing the one-dimensional case of a seesaw.
- **(Section 15.4)** Developed the equation of the tangent plane in a manner that makes geometric sense.
- **(Section 15.5)** Included a proof of the fact the gradient of a function f of three variables is orthogonal to the surfaces that are the level sets of f.
- **(Section 15.8)** Gave an intuitive explanation for why the Method of Lagrange Multipliers works.

REORDERING AND ADDING TOPICS There were some specific rearrangements among the sections and additions. These include:

- A subsection on piecewise-defined functions has been added to Section 1.3.
- The section on indefinite integrals (previously Section 4.8) has been moved from Chapter 4 (Applications of the Derivative) to Chapter 5 (The Integral). This is a more natural placement for it.
- A new section on choosing from amongst the various methods of integration has been added to Chapter 8.
- A subsection on choosing the appropriate convergence/divergence test has been added to Section 11.5.
- An explanation of how to find indefinite limits using power series has been added to Section 10.6.
- The definitions of divergence and curl have been moved from Chapter 18 to Section 17.1. This allows us to utilize them at an appropriate earlier point in the text.
- A list all of the different types of integrals that have been introduced in Chapter 17 has been added to Section 17.5.
- A subsection on the Vector Form of Green's Theorem has been added to Section 18.1.

NEW EXAMPLES, FIGURES, AND EXERCISES Numerous examples and accompanying figures have been added to clarify concepts. A variety of exercises have also been added throughout the text, particularly where new applications are available or further conceptual development is advantageous. Figures marked with a DF icon have been made dynamic and can be accessed via *LaunchPad*. A selection of these figures also includes brief tutorial videos explaining the concepts at work.

SUPPLEMENTS

For Instructors

Instructor's Solutions Manual
Contains worked-out solutions to all exercises in the text.

Test Bank
Includes a comprehensive set of multiple-choice test items.

Instructor's Resource Manual
Provides sample course outlines, suggested class time, key points, lecture material, discussion topics, class activities, worksheets, projects, and questions to accompany the Dynamic Figures.

For Students

Student Solutions Manual
Single Variable ISBN: 1-4641-7503-9
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Contains worked-out solutions to all odd-numbered exercises in the text.

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Our new course space, LaunchPad, combines an interactive e-Book with high-quality multimedia content and ready-made assessment options, including LearningCurve adaptive quizzing. Pre-built, curated units are easy to assign or adapt with your own material, such as readings, videos, quizzes, discussion groups, and more. LaunchPad includes a gradebook that provides a clear window on performance for your whole class, for individual students, and for individual assignments. While a streamlined interface helps students focus on what's due next, social commenting tools let them engage, make connections, and learn from each other. Use LaunchPad on its own or integrate it with your school's learning management system so your class is always on the same page. Contact your rep to make sure you have access.

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Interactive e-Book: Every LaunchPad e-Book comes with powerful study tools for students, video and multimedia content, and easy customization for instructors. Students can search, highlight, and bookmark, making it easier to study and access key content. And instructors can make sure their class gets just the book they want to deliver: customize and rearrange chapters, add and share notes and discussions, and link to quizzes, activities, and other resources.

Macmillan Education LearningCurve

LearningCurve provides students and instructors with powerful adaptive quizzing, a game-like format, direct links to the e-Book, and instant feedback. The quizzing system features questions tailored specifically to the text and adapts to students' responses, providing material at different difficulty levels and topics based on student performance.

Dynamic Figures: Over 250 figures from the text have been recreated in a new interactive format for students and instructors to manipulate and explore, making the visual aspects and dimensions of calculus concepts easier to grasp. Brief tutorial videos accompany selected figures and explain the concepts at work.

CalcClips: These whiteboard tutorials provide animated and narrated step-by-step solutions to exercises that are based on key problems in the text.

Freeman SolutionMaster

SolutionMaster offers an easy-to-use Web-based version of the instructor's solutions, allowing instructors to generate a solution file for any set of homework exercises.

www.webassign.net/freeman.com

WebAssign Premium integrates the book's exercises into the world's most popular and trusted online homework system, making it easy to assign algorithmically generated homework and quizzes. Algorithmic exercises offer the instructor optional algorithmic solutions. WebAssign Premium also offers access to resources, including Dynamic Figures, CalcClips whiteboard tutorials, and a "Show My Work" feature. In addition, WebAssign Premium is available with a fully customizable e-Book option.

webwork.maa.org

W. H. Freeman offers thousands of algorithmically generated questions (with full solutions) through this free, open-source online homework system created at the University of Rochester. Adopters also have access to a shared national library test bank with thousands of additional questions, including 2,500 problem sets matched to the book's table of contents.

FEATURES

Conceptual Insights

encourage students to develop a conceptual understanding of calculus by explaining important ideas clearly but informally.

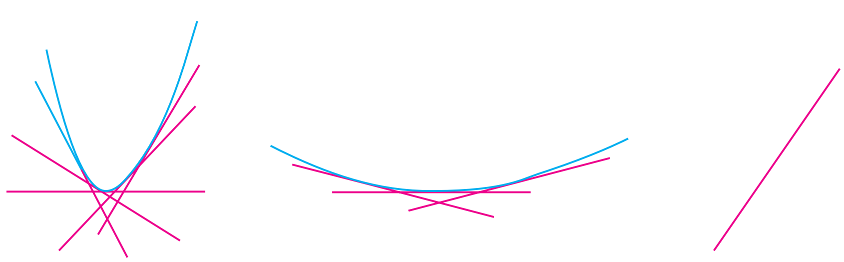
CONCEPTUAL INSIGHT Leibniz notation is widely used for several reasons. First, it reminds us that the derivative df/dx, although not itself a ratio, is in fact a *limit* of ratios $\Delta f/\Delta x$. Second, the notation specifies the independent variable. This is useful when variables other than x are used. For example, if the independent variable is t, we write df/dt. Third, we often think of d/dx as an "operator" that performs differentiation on functions. In other words, we apply the operator d/dx to f to obtain the derivative df/dx. We will see other advantages of Leibniz notation when we discuss the Chain Rule in Section 3.7.

Ch. 3, p. 107

Graphical Insights

enhance students' visual understanding by making the crucial connections between graphical properties and the underlying concepts.

GRAPHICAL INSIGHT Can we visualize the rate represented by $f''(x)$? The second derivative is the rate at which $f'(x)$ is changing, so $f''(x)$ is large if the slopes of the tangent lines change rapidly, as in Figure 3(A). Similarly, $f''(x)$ is small if the slopes of the tangent lines change slowly—in this case, the curve is relatively flat, as in Figure 3(B). If f is a linear function [Figure 3(C)], then the tangent line does not change at all and $f''(x) = 0$. Thus, $f''(x)$ measures the "bending" or concavity of the graph.



(A) Large second derivative: Tangent lines turn rapidly.

(B) Smaller second derivative: Tangent lines turn slowly.

(C) Second derivative is zero: Tangent line does not change.

DF **FIGURE 3**

Ch. 3, p. 134

Reminders

are margin notes that link the current discussion to important concepts introduced earlier in the text to give students a quick review and make connections with related ideas.

■ **EXAMPLE 3** Evaluate $\int \sin^2 x\,dx$.

Solution We could apply the reduction formula Eq. (5) from the last section. However, instead, we apply a method that does not rely on knowing that formula. We utilize the trigonometric identity called the double angle formula $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$. Then

$$\int \sin^2 x\,dx = \int \frac{1}{2}(1 - \cos 2x)\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$ ■

REMINDER Useful Identities:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

Using the trigonometric identities in the margin, we can also integrate $\cos^2 x$, obtaining the following:

$$\boxed{\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C = \frac{x}{2} - \frac{1}{2} \sin x \cos x + C} \qquad \boxed{1}$$

$$\boxed{\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C = \frac{x}{2} + \frac{1}{2} \sin x \cos x + C} \qquad \boxed{2}$$

Ch. 8, p. 398

Caution Notes

warn students of common pitfalls they may encounter in understanding the material.

CAUTION When using L'Hôpital's Rule, be sure to take the derivative of the numerator and denominator separately:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

Do not differentiate the quotient function $y = f(x)/g(x)$.

■ **EXAMPLE 1** Use L'Hôpital's Rule to evaluate $\lim_{x\to 2} \frac{x^3 - 8}{x^4 + 2x - 20}$.

Solution Let $f(x) = x^3 - 8$ and $g(x) = x^4 + 2x - 20$. Both f and g are differentiable and $f(x)/g(x)$ is indeterminate of type $0/0$ at $a = 2$ because $f(2) = g(2) = 0$:

- Numerator: $f(2) = 2^3 - 1 = 0$
- Denominator: $g(2) = 2^4 + 2(2) - 20 = 0$

Furthermore, $g'(x) = 4x^3 + 2$ is nonzero near $x = 2$, so L'Hôpital's Rule applies. We may replace the numerator and denominator by their derivatives to obtain

$$\underbrace{\lim_{x\to 2} \frac{x^3 - 8}{x^4 + 2x - 2} = \lim_{x\to 2} \frac{(x^3 - 8)'}{(x^4 + 2x - 2)'}}_{\text{L'Hôpital's Rule}} = \lim_{x\to 2} \frac{3x^2}{4x^3 + 2} = \frac{3(2^2)}{4(2^3) + 2} = \frac{12}{34} = \frac{6}{17}$$ ■

Ch. 7, p. 362

Historical Perspectives

are brief vignettes that place key discoveries and conceptual advances in their historical context. They give students a glimpse into some of the accomplishments of great mathematicians and an appreciation for their significance.

(Mechanics Magazine London, 1824)

HISTORICAL PERSPECTIVE

Geometric series were used as early as the third century BCE by Archimedes in a brilliant argument for determining the area S of a "parabolic segment" (shaded region in Figure 3). Given two points A and C on a parabola, there is a point B between A and C where the tangent line is parallel to $\overline{AC}$ (apparently, Archimedes was aware of the Mean Value Theorem more than 2000 years before the invention of calculus). Let T be the area of triangle $\triangle ABC$. Archimedes proved that if D is chosen in a similar fashion relative to $\overline{AB}$ and E is chosen relative to $\overline{BC}$, then

$$\frac{1}{4}T = \text{Area}(\triangle ADB) + \text{Area}(\triangle BEC) \qquad \boxed{6}$$

This construction of triangles can be continued. The next step would be to construct the four triangles on the segments $\overline{AD}$, $\overline{DB}$, $\overline{BE}$, $\overline{EC}$, of total area $\frac{1}{4}^2 T$. Then construct eight triangles of total area $\frac{1}{4}^3 T$, etc. In this way, we obtain infinitely many triangles that completely fill up the parabolic segment. By the formula for the sum of a geometric series, we get

$$S = T + \frac{1}{4}T + \frac{1}{16}T + \cdots = T \sum_{n=0}^{\infty} \frac{1}{4^n} = \frac{4}{3}T$$

For this and many other achievements, Archimedes is ranked together with Newton and Gauss as one of the greatest scientists of all time.

The modern study of infinite series began in the seventeenth century with Newton, Leibniz, and their contemporaries. The divergence of $\sum_{n=1}^{\infty} 1/n$ (called the **harmonic series**) was known to the medieval scholar Nicole d'Oresme (1323–1382), but his proof was lost for centuries, and the result was rediscovered on more than one occasion. It was also known that the sum of the reciprocal squares $\sum_{n=1}^{\infty} 1/n^2$ converges, and in the 1640s, the Italian Pietro Mengoli put forward the challenge of finding its sum. Despite the efforts of the best mathematicians of the day, including Leibniz and the Bernoulli brothers Jakob and Johann, the problem resisted solution for nearly a century. In 1735 the great master Leonhard Euler (at the time, 28 years old) astonished his contemporaries by proving that

$$\boxed{\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6}}$$

This formula, surprising in itself, plays a role in a variety of mathematical fields. A theorem from number theory states that two whole numbers, chosen randomly, have no common factor with probability $6/\pi^2 \approx 0.6$ (the reciprocal of Euler's result). On the other hand, Euler's result and its generalizations appear in the field of statistical mechanics.

Ch. 11, p. 546

Assumptions Matter uses short explanations and well-chosen counterexamples to help students appreciate why hypotheses are needed in theorems.

■ **EXAMPLE 3** **Assumptions Matter** Show that the Product Law cannot be applied to $\lim_{x\to 0} f(x)g(x)$ if $f(x) = x$ and $g(x) = x^{-1}$.

Solution For all $x \neq 0$, we have $f(x)g(x) = x \cdot x^{-1} = 1$, so the limit of the product exists:

$$\lim_{x\to 0} f(x)g(x) = \lim_{x\to 0} 1 = 1$$

However, $\lim_{x\to 0} x^{-1}$ does not exist because $g(x) = x^{-1}$ approaches ∞ as $x \to 0^+$ and it approaches $-\infty$ as $x \to 0^-$. Therefore, the Product Law cannot be applied and its conclusion does not hold:

$$\left(\lim_{x\to 0} f(x)\right)\left(\lim_{x\to 0} g(x)\right) = \left(\lim_{x\to 0} x\right)\underbrace{\left(\lim_{x\to 0} x^{-1}\right)}_{\text{Does not exist}}$$ ■

Ch. 2, p. 58

Section Summaries summarize a section's key points in a concise and useful way and emphasize for students what is most important in each section.

Section Exercise Sets offer a comprehensive set of exercises closely coordinated with the text. These exercises vary in difficulty from routine, to moderate, to more challenging. Also included are icons indicating problems that require the student to give a written response or require the use of technology T.

Chapter Review Exercises offer a comprehensive set of exercises closely coordinated with the chapter material to provide additional problems for self-study or assignments.

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Colin Adams

Functions that yield the amount of seismic activity as a function of time help scientists to predict volcanic eruptions and earthquakes.

(*Douglas Peebles/Science Source*)

1 PRECALCULUS REVIEW

Calculus builds on the foundation of algebra, analytic geometry, and trigonometry. In this chapter, therefore, we review some concepts, facts, and formulas from precalculus that are used throughout the text. In the last section, we discuss ways in which technology can be used to enhance your visual understanding of functions and their properties.

1.1 Real Numbers, Functions, and Graphs

We begin with a short discussion of real numbers. This gives us the opportunity to recall some basic properties and standard notation.

A **real number** is a number represented by a decimal or "decimal expansion." There are three types of decimal expansions: finite, repeating, and infinite but nonrepeating. For example,

$$\frac{3}{8} = 0.375, \qquad \frac{1}{7} = 0.142857142857\ldots = 0.\overline{142857}$$

$$\pi = 3.141592653589793\ldots$$

The number $\frac{3}{8}$ is represented by a finite decimal, whereas $\frac{1}{7}$ is represented by a *repeating* or *periodic* decimal. The bar over 142857 indicates that this sequence repeats indefinitely. The decimal expansion of π is infinite but nonrepeating.

The set of all real numbers is denoted by a boldface **R**. When there is no risk of confusion, we refer to a real number simply as a *number*. We also use the standard symbol $\in$ for the phrase "belongs to." Thus,

$$a \in \mathbf{R} \qquad \text{reads} \qquad \text{“}a \text{ belongs to } \mathbf{R}\text{”}$$

Additional properties of real numbers are discussed in Appendix B.

The set of integers is commonly denoted by the letter **Z** (this choice comes from the German word *Zahl*, meaning "number"). Thus, $\mathbf{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$. A **whole number** is a nonnegative integer—that is, one of the numbers $0, 1, 2, \ldots$.

A real number is called **rational** if it can be represented by a fraction p/q, where p and q are integers with $q \neq 0$. The set of rational numbers is denoted **Q** (for "quotient"). Numbers that are not rational, such as π and $\sqrt{2}$, are called **irrational**.

We can tell whether a number is rational from its decimal expansion: Rational numbers have finite or repeating decimal expansions, and irrational numbers have infinite, nonrepeating decimal expansions. Furthermore, the decimal expansion of a number is unique, apart from the following exception: Every finite decimal is equal to an infinite decimal in which the digit 9 repeats. For example,

$$1 = 0.999\ldots, \qquad \frac{3}{8} = 0.375 = 0.374999\ldots, \qquad \frac{47}{20} = 2.35 = 2.34999\ldots$$

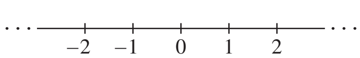


FIGURE 1 The set of real numbers represented as a line.

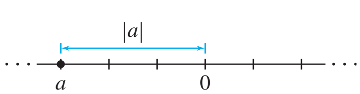


FIGURE 2 $|a|$ is the distance from a to the origin.

We visualize real numbers as points on a line (Figure 1). For this reason, real numbers are often referred to as **points**. The point corresponding to 0 is called the **origin**.

The **absolute value** of a real number a, denoted $|a|$, is defined by (Figure 2)

$$|a| = \textit{distance from the origin} = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

For example, $|1.2| = 1.2$ and $|-8.35| = 8.35$. The absolute value satisfies

$$\boxed{|a| = |-a|, \qquad |ab| = |a|\,|b|}$$

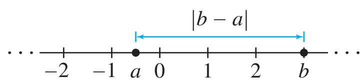


FIGURE 3 The distance from a to b is $|b - a|$.

The **distance** between two real numbers a and b is $|b - a|$, which is the length of the line segment joining a and b (Figure 3).

Two real numbers a and b are close to each other if $|b - a|$ is small, and this is the case if their decimal expansions agree to many places. More precisely, *if the decimal expansions of a and b agree to k places (to the right of the decimal point), then the distance $|b - a|$ is at most* 10^{-k}. Thus, the distance between $a = 3.1415$ and $b = 3.1478$ is at most 10^{-2} because a and b agree to two places. In fact, the distance is exactly $|3.1478 - 3.1415| = 0.0063$.

Beware that $|a + b|$ is not equal to $|a| + |b|$ unless a and b have the same sign or at least one of a and b is zero. If they have opposite signs, cancellation occurs in the sum $a + b$, and $|a + b| < |a| + |b|$. For example, $|2 + 5| = |2| + |5|$ but $|-2 + 5| = 3$, which is less than $|-2| + |5| = 7$. In any case, $|a + b|$ is never larger than $|a| + |b|$ and this gives us the simple but important **triangle inequality**:

$$\boxed{|a + b| \le |a| + |b|} \qquad \boxed{1}$$

We use standard notation for intervals. Given real numbers $a < b$, there are four intervals with endpoints a and b (Figure 4). They all have length $b - a$ but differ according to which endpoints are included.

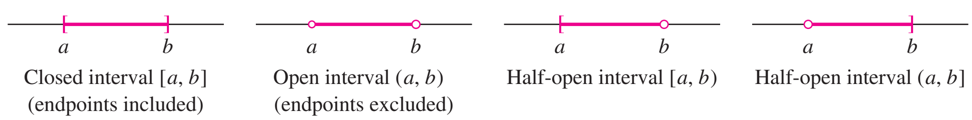


FIGURE 4 The four intervals with endpoints a and b.

The **closed interval** $[a, b]$ is the set of all real numbers x such that $a \le x \le b$:

$$[a, b] = \{x \in \mathbf{R} : a \le x \le b\}$$

The notation $(2, 3)$ could mean the open interval $\{x : 2 < x < 3\}$ or it could mean the point in the xy-plane with $x = 2$ and $y = 3$. In general, the meaning will be apparent from the context.

We usually write this more simply as $\{x : a \le x \le b\}$, it being understood that x belongs to $\mathbf{R}$. The **open** and **half-open intervals** are the sets

$$\underbrace{(a, b) = \{x : a < x < b\}}_{\text{Open interval (endpoints excluded)}}, \qquad \underbrace{[a, b) = \{x : a \le x < b\}}_{\text{Half-open interval}}, \qquad \underbrace{(a, b] = \{x : a < x \le b\}}_{\text{Half-open interval}}$$

The infinite interval $(-\infty, \infty)$ is the entire real line $\mathbf{R}$. A half-infinite interval is closed if it contains its finite endpoint and is open otherwise (Figure 5):

$$[a, \infty) = \{x : a \le x\}, \qquad (-\infty, b] = \{x : x \le b\}$$

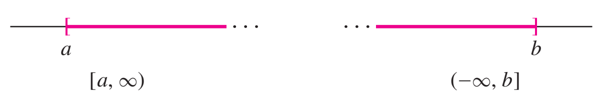


FIGURE 5 Closed half-infinite intervals.

Open and closed intervals may be described by inequalities. For example, the interval $(-r, r)$ is described by the inequality $|x| < r$ (Figure 6):

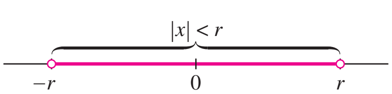


FIGURE 6 The interval $(-r, r) = \{x : |x| < r\}$.

$$\boxed{|x| < r \quad \Leftrightarrow \quad -r < x < r \quad \Leftrightarrow \quad x \in (-r, r)} \qquad \boxed{2}$$

More generally, for an interval symmetric about the value c (Figure 7),

$$\boxed{|x - c| < r \quad \Leftrightarrow \quad c - r < x < c + r \quad \Leftrightarrow \quad x \in (c - r, c + r)} \qquad \boxed{3}$$

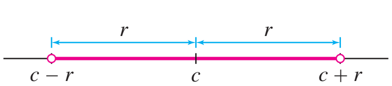


FIGURE 7 $(a, b) = (c - r, c + r)$, where

$$c = \frac{a + b}{2}, \qquad r = \frac{b - a}{2}$$

Closed intervals are similar, with $<$ replaced by $\le$. We refer to r as the **radius** and to c as the **midpoint** or **center**. The intervals (a, b) and $[a, b]$ have midpoint $c = \frac{1}{2}(a + b)$ and radius $r = \frac{1}{2}(b - a)$ (Figure 7).

■ **EXAMPLE 1** Describe [7, 13] using inequalities.

Solution The midpoint of the interval [7, 13] is $c = \frac{1}{2}(7 + 13) = 10$ and its radius is $r = \frac{1}{2}(13 - 7) = 3$ (Figure 8). Therefore,

$$[7, 13] = \{x \in \mathbf{R} : |x - 10| \le 3\}$$ ■

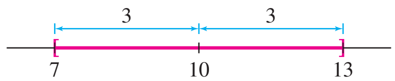


FIGURE 8 The interval [7, 13] is described by $|x - 10| \le 3$.

■ **EXAMPLE 2** Describe the set $S = \left\{x : \left|\frac{1}{2}x - 3\right| > 4\right\}$ in terms of intervals.

Solution It is easier to consider the opposite inequality $\left|\frac{1}{2}x - 3\right| \le 4$ first. By (2),

$$\left|\frac{1}{2}x - 3\right| \le 4 \quad \Leftrightarrow \quad -4 \le \frac{1}{2}x - 3 \le 4$$

$$-1 \le \frac{1}{2}x \le 7 \qquad \text{(add 3)}$$

$$-2 \le x \le 14 \qquad \text{(multiply by 2)}$$

In Example 2 we use the notation ∪ to denote "union": The union $A \cup B$ of sets A and B consists of all elements that belong to either A or B (or to both).

Thus, $\left|\frac{1}{2}x - 3\right| \le 4$ is satisfied when x belongs to $[-2, 14]$. The set S is the *complement*, consisting of all numbers x *not in* $[-2, 14]$. We can describe S as the union of two intervals: $S = (-\infty, -2) \cup (14, \infty)$ (Figure 9). ■

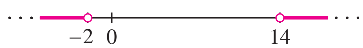


FIGURE 9 The set $S = \left\{x : \left|\frac{1}{2}x - 3\right| > 4\right\}$.

Graphing

Graphing is a basic tool in calculus, as it is in algebra and trigonometry. Recall that rectangular (or Cartesian) coordinates in the plane are defined by choosing two perpendicular axes, the x-axis and the y-axis.To a pair of numbers (a, b) we associate the point P located at the intersection of the line perpendicular to the x-axis at a and the line perpendicular to the y-axis at b [Figure 10(A)]. The numbers a and b are the x- and y-**coordinates** of P. The x-coordinate is sometimes called the "abscissa" and the y-coordinate the "ordinate." The **origin** is the point with coordinates (0, 0).

The term "Cartesian" refers to the French philosopher and mathematician René Descartes (1596–1650), whose Latin name was Cartesius. He is credited (along with Pierre de Fermat) with the invention of analytic geometry. In his great work La Géométrie, *Descartes used the letters x, y, z for unknowns and a, b, c for constants, a convention that has been followed ever since.*

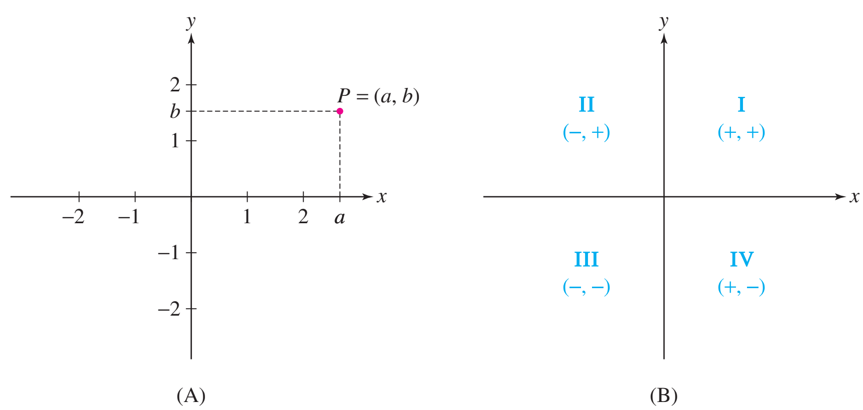


FIGURE 10 Rectangular coordinate system.

The axes divide the plane into four quadrants labeled I–IV, determined by the signs of the coordinates [Figure 10(B)]. For example, quadrant III consists of points (x, y) such that $x < 0$ and $y < 0$.

The distance d between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is computed using the Pythagorean Theorem. In Figure 11, we see that $\overline{P_1 P_2}$ is the hypotenuse of a right triangle with sides $a = |x_2 - x_1|$ and $b = |y_2 - y_1|$. Therefore,

$$d^2 = a^2 + b^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

We obtain the distance formula by taking square roots.

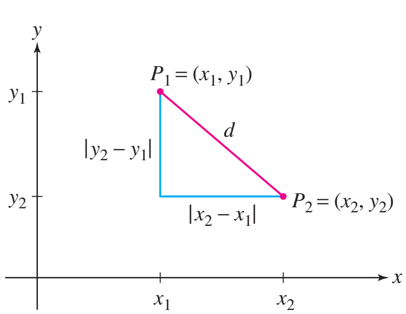


FIGURE 11 Distance d is given by the distance formula.

Distance Formula The distance between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is equal to

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

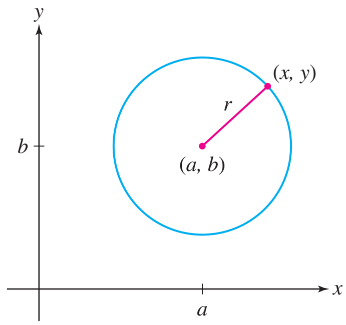


FIGURE 12 Circle with equation $(x-a)^2 + (y-b)^2 = r^2$.

Once we have the distance formula, we can derive the equation of a circle of radius r and center (a, b) (Figure 12). A point (x, y) lies on this circle if the distance from (x, y) to (a, b) is r:

$$\sqrt{(x-a)^2 + (y-b)^2} = r$$

Squaring both sides, we obtain the standard equation of the circle:

$$\boxed{(x-a)^2 + (y-b)^2 = r^2}$$

We now review some definitions and notation concerning functions.

DEFINITION A **function** f from a set D to a set Y is a rule that assigns, to each element x in D, a unique element $y = f(x)$ in Y. We write

$$f : D \to Y$$

The set D, called the **domain** of f, is the set of "allowable inputs." For $x \in D$, $f(x)$ is called the **value** of f at x (Figure 13). The **range** R of f is the subset of Y consisting of all values $f(x)$:

$$R = \{y \in Y : f(x) = y \text{ for some } x \in D\}$$

A function $f : D \to Y$ is also called a "map." The sets D and Y can be arbitrary. For example, we can define a map from the set of living people to the set of whole numbers by mapping each person to his or her year of birth. The range of this map is the set of years in which a living person was born. In multivariable calculus, the domain might be a set of points in the two-dimensional plane and the range a set of numbers, points, or vectors.

Informally, we think of f as a "machine" that produces an output y for every input x in the domain D (Figure 14).

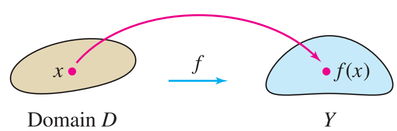


FIGURE 13 A function assigns an element $f(x)$ in Y to each $x \in D$.

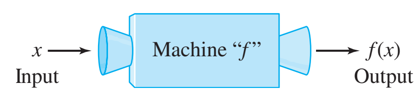


FIGURE 14 Think of f as a "machine" that takes the input x and produces the output $f(x)$.

The first part of this text deals with *numerical* functions f, where both the domain and the range are sets of real numbers. We refer to such a function as f and its value at x as $f(x)$. The letter x is used often to denote the **independent variable** that can take on any value in the domain D. We write $y = f(x)$ and refer to y as the **dependent variable** (because its value depends on the choice of x).

When f is defined by a formula, its natural domain is the set of real numbers x for which the formula is meaningful. For example, the function $f(x) = \sqrt{9-x}$ has domain $D = \{x : x \le 9\}$ because $\sqrt{9-x}$ is defined if $9 - x \ge 0$. Here are some other examples of domains and ranges:

$f(x)$	Domain D	Range R
x^2	$\mathbf{R}$	$\{y : y \ge 0\}$
$\cos x$	$\mathbf{R}$	$\{y : -1 \le y \le 1\}$
$\dfrac{1}{x+1}$	$\{x : x \ne -1\}$	$\{y : y \ne 0\}$

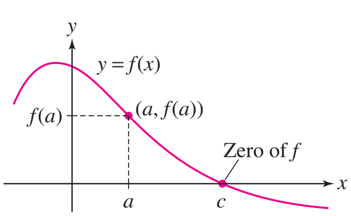


FIGURE 15

The **graph** of a function $y = f(x)$ is obtained by plotting the points $(a, f(a))$ for a in the domain D (Figure 15). If you start at $x = a$ on the x-axis, move up to the graph and then over to the y-axis, you arrive at the value $f(a)$. The absolute value $|f(a)|$ is the distance from the graph to the x-axis.

A **zero** or **root** of a function f is a number c such that $f(c) = 0$. The zeros are the values of x where the graph intersects the x-axis.

In Chapter 4, we will use calculus to sketch and analyze graphs. At this stage, to sketch a graph by hand, we can make a table of function values, plot the corresponding points (including any zeros), and connect them by a smooth curve.

EXAMPLE 3 Find the roots and sketch the graph of $f(x) = x^3 - 2x$.

Solution First, we solve

$$x^3 - 2x = x(x^2 - 2) = 0$$

The roots of f are $x = 0$ and $x = \pm\sqrt{2}$. To sketch the graph, we plot the roots and a few values listed in Table 1 and join them by a curve (Figure 16).

TABLE 1

x	$x^3 - 2x$
-2	-4
-1	1
0	0
1	-1
2	4

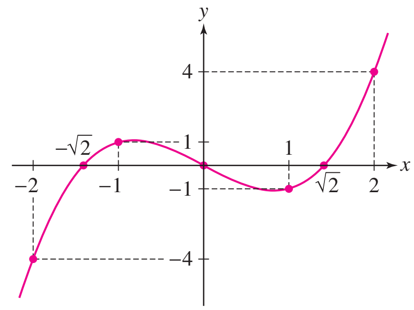


FIGURE 16 Graph of $f(x) = x^3 - 2x$.

Functions arising in applications are not always given by formulas. For example, data collected from observation or experiment define functions for which there may be no exact formula. Such functions can be displayed either graphically or by a table of values. Figure 17 and Table 2 display data collected by biologist Julian Huxley (1887–1975) in a study of the antler weight W of male red deer as a function of age t. We will see that many of the tools from calculus can be applied to functions constructed from data in this way.

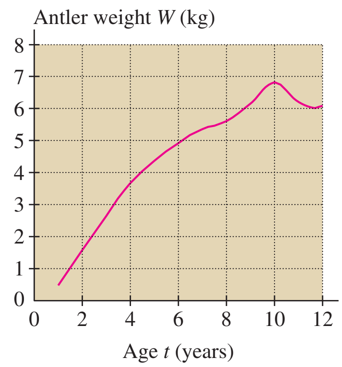


FIGURE 17 Male red deer shed their antlers every winter and regrow them in the spring. This graph shows average antler weight as a function of age.

TABLE 2

t (years)	W (kg)	t (years)	W (kg)
1	0.48	7	5.34
2	1.59	8	5.62
3	2.66	9	6.18
4	3.68	10	6.81
5	4.35	11	6.21
6	4.92	12	6.1

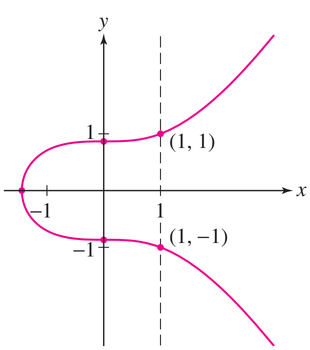


FIGURE 18 Graph of $4y^2 - x^3 = 3$. This graph fails the Vertical Line Test, so it is not the graph of a function.

We can graph not just functions but, more generally, any equation relating y and x. Figure 18 shows the graph of the equation $4y^2 - x^3 = 3$; it consists of all pairs (x, y) satisfying the equation. This curve is not the graph of a function because some x-values are associated with two y-values. For example, $x = 1$ is associated with $y = \pm 1$. A curve is the graph of a function if and only if it passes the **Vertical Line Test**; that is, every vertical line $x = a$ intersects the curve in at most one point.

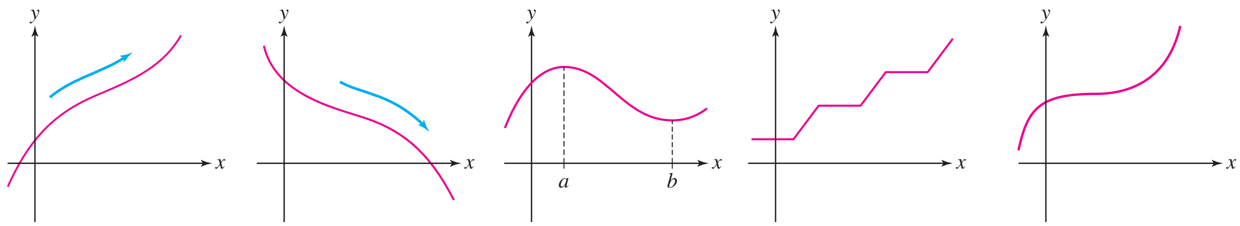
We are often interested in whether a function is increasing or decreasing. Roughly speaking, a function f is increasing if its graph goes up as we move to the right and is decreasing if its graph goes down [Figures 19(A) and (B)]. More precisely, we define the notion of increase/decrease on an open interval.

A function f is:

- **increasing** on (a, b) if $f(x_1) < f(x_2)$ for all $x_1, x_2 \in (a, b)$ such that $x_1 < x_2$.
- **decreasing** on (a, b) if $f(x_1) > f(x_2)$ for all $x_1, x_2 \in (a, b)$ such that $x_1 < x_2$.

We say that f is **monotonic** if it is either increasing or decreasing. In Figure 19(C), the function is not monotonic because it is neither increasing nor decreasing for all x.

A function f is called **nondecreasing** if $f(x_1) \le f(x_2)$ for $x_1 < x_2$ (defined by $\le$ rather than a strict inequality $<$). **Nonincreasing** functions are defined similarly. Function (D) in Figure 19 is nondecreasing, but it is not increasing on the intervals where the graph is horizontal. Function (E) is increasing everywhere even though it levels off momentarily.



(A) Increasing (B) Decreasing (C) Decreasing on (a, b) but not decreasing everywhere (D) Nondecreasing but not increasing (E) Increasing

FIGURE 19

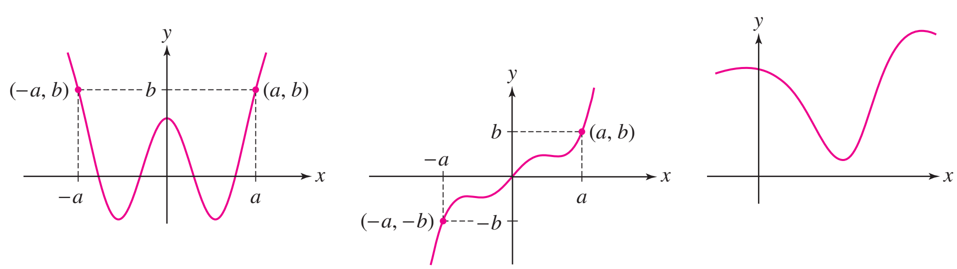
Another important property is **parity**, which refers to whether a function is even or odd:

- f is **even** if $f(-x) = f(x)$
- f is **odd** if $f(-x) = -f(x)$

The graphs of functions with even or odd parity have a special symmetry:

- **Even function:** Graph is symmetric about the y-axis. This means that if $P = (a, b)$ lies on the graph, then so does $Q = (-a, b)$ [Figure 20(A)].
- **Odd function:** Graph is symmetric with respect to the origin. This means that if $P = (a, b)$ lies on the graph, then so does $Q = (-a, -b)$ [Figure 20(B)].

Many functions are neither even nor odd [Figure 20(C)].



(A) Even function: $f(-x) = f(x)$ Graph is symmetric about the y-axis.

(B) Odd function: $f(-x) = -f(x)$ Graph is symmetric about the origin.

(C) Neither even nor odd

FIGURE 20

■ **EXAMPLE 4** Determine whether the function is even, odd, or neither.

(a) $f(x) = x^4$ **(b)** $g(x) = x^{-1}$ **(c)** $h(x) = x^2 + x$

Solution

(a) $f(-x) = (-x)^4 = x^4$. Thus, $f(x) = f(-x)$, and f is even.

(b) $g(-x) = (-x)^{-1} = -x^{-1}$. Thus, $g(-x) = -g(x)$, and g is odd.

(c) $h(-x) = (-x)^2 + (-x) = x^2 - x$. We see that $h(-x)$ is not equal to $h(x)$ or to $-h(x) = -x^2 - x$. Therefore, h is neither even nor odd. ■

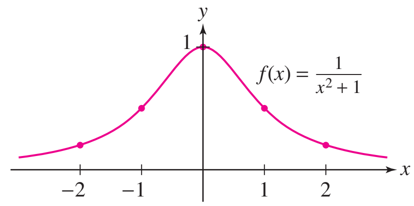
■ **EXAMPLE 5 Using Symmetry** Sketch the graph of $f(x) = \dfrac{1}{x^2 + 1}$.

Solution The function f is positive $[f(x) > 0]$ and even $[f(-x) = f(x)]$. Therefore, the graph lies above the x-axis and is symmetric with respect to the y-axis. Furthermore,

f is decreasing for $x \geq 0$ (because a larger value of x makes the denominator larger). We use this information and a short table of values (Table 3) to sketch the graph (Figure 21). Note that the graph approaches the x-axis as we move to the right or left because $f(x)$ gets closer to 0 as $|x|$ increases. ■

TABLE 3

x	$\dfrac{1}{x^2+1}$
0	1
±1	$\frac{1}{2}$
±2	$\frac{1}{5}$



DF **FIGURE 21**

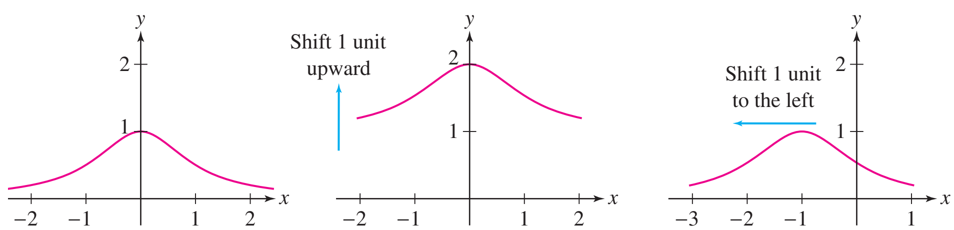
Two important ways of modifying a graph are **translation** (or **shifting**) and **scaling**. Translation consists of moving the graph horizontally or vertically:

Remember that $f(x) + c$ and $f(x + c)$ are different. The graph of $y = f(x) + c$ is a vertical translation and $y = f(x + c)$ a horizontal translation of the graph of $y = f(x)$.

DEFINITION Translation (Shifting)

- **Vertical translation** $y = f(x) + c$: Shifts the graph by $|c|$ units *vertically*, upward if $c > 0$ and downward if $c < 0$.
- **Horizontal translation** $y = f(x + c)$: Shifts the graph by $|c|$ units *horizontally*, to the right if $c < 0$ and c units to the left if $c > 0$.

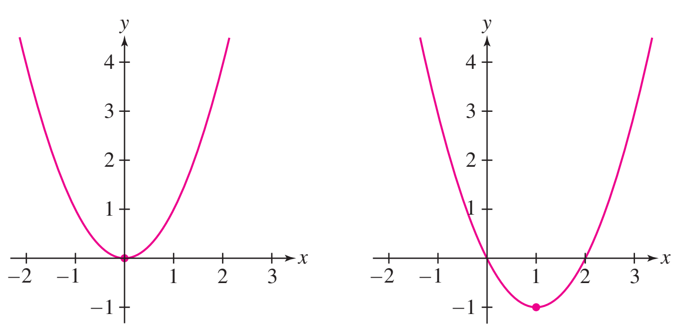
Figure 22 shows the effect of translating the graph of $f(x) = 1/(x^2 + 1)$ vertically and horizontally.



(A) $y = f(x) = \frac{1}{x^2+1}$ (B) $y = f(x) + 1 = \frac{1}{x^2+1} + 1$ (C) $y = f(x+1) = \frac{1}{(x+1)^2+1}$

FIGURE 22

■ **EXAMPLE 6** Figure 23(A) is the graph of $f(x) = x^2$, and Figure 23(B) is a horizontal and vertical shift of (A). What is the equation of graph (B)?



(A) $f(x) = x^2$ (B)

DF **FIGURE 23**

Solution Graph (B) is obtained by shifting graph (A) 1 unit to the right and 1 unit down. We can see this by observing that the point (0, 0) on the graph of f is shifted to (1, −1). Therefore, (B) is the graph of $g(x) = (x - 1)^2 - 1$. ■

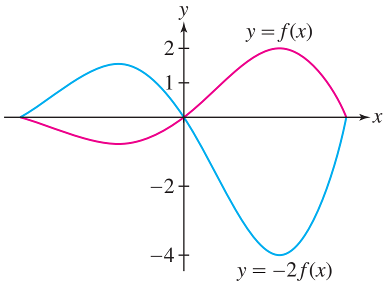


FIGURE 24 Negative vertical scale factor $k = -2$.

Scaling (also called **dilation**) consists of compressing or expanding the graph in the vertical or horizontal directions:

DEFINITION Scaling

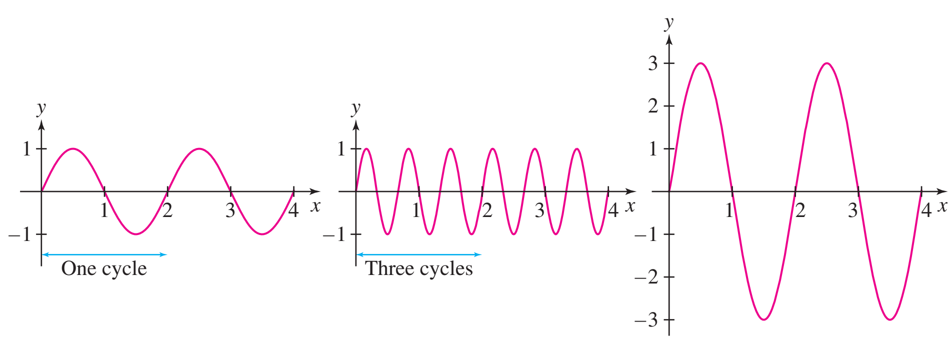
- **Vertical scaling** $y = kf(x)$: If $k > 1$, the graph is expanded vertically by the factor k. If $0 < k < 1$, the graph is compressed vertically. When the scale factor k is negative ($k < 0$), the graph is also reflected across the x-axis (Figure 24).
- **Horizontal scaling** $y = f(kx)$: If $k > 1$, the graph is compressed in the horizontal direction. If $0 < k < 1$, the graph is expanded. If $k < 0$, then the graph is also reflected across the y-axis.

The amplitude of a function is half the difference between its greatest value and its least value, if it has both a greatest value and least value. Thus, vertical scaling changes the amplitude by the factor $|k|$.

EXAMPLE 7 Sketch the graphs of $f(x) = \sin(\pi x)$ and its dilates $f(3x)$ and $3f(x)$.

Solution The graph of $f(x) = \sin(\pi x)$ is a sine curve with period 2. It completes one cycle over every interval of length 2—see Figure 25(A). It has amplitude 1.

- The graph of $f(3x) = \sin(3\pi x)$ is a compressed version of $y = f(x)$, completing three cycles instead of one over intervals of length 2 [Figure 25(B)]. It also has amplitude 1.
- The graph of $y = 3f(x) = 3\sin(\pi x)$ differs from $y = f(x)$ only in amplitude: It is expanded in the vertical direction by a factor of 3 [Figure 25(C)], so its amplitude is 3. ■



(A) $y = f(x) = \sin(\pi x)$

(B) Horizontal compression: $y = f(3x) = \sin(3\pi x)$

(C) Vertical expansion: $y = 3f(x) = 3\sin(\pi x)$

DF **FIGURE 25** Horizontal and vertical scaling of $f(x) = \sin(\pi x)$.

1.1 SUMMARY

- Absolute value: $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$
- Triangle inequality: $|a + b| \leq |a| + |b|$
- Four intervals with endpoints a and b:

$$(a, b), \qquad [a, b], \qquad [a, b), \qquad (a, b]$$

- Writing open and closed intervals using inequalities:

$$(a, b) = \{x : |x - c| < r\}, \qquad [a, b] = \{x : |x - c| \leq r\}$$

where $c = \frac{1}{2}(a + b)$ is the midpoint and $r = \frac{1}{2}(b - a)$ is the radius.

- Distance d between (x_1, y_1) and (x_2, y_2):

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Equation of circle of radius r with center (a, b):

$$(x - a)^2 + (y - b)^2 = r^2$$

- A *zero* or *root* of a function f is a number c such that $f(c) = 0$.
- Vertical Line Test: A curve in the plane is the graph of a function if and only if each vertical line $x = a$ intersects the curve in at most one point.
-

Increasing:	$f(x_1) < f(x_2)$ if $x_1 < x_2$
Nondecreasing:	$f(x_1) \le f(x_2)$ if $x_1 < x_2$
Decreasing:	$f(x_1) > f(x_2)$ if $x_1 < x_2$
Nonincreasing:	$f(x_1) \ge f(x_2)$ if $x_1 < x_2$

- Even function: $f(-x) = f(x)$ (graph is symmetric about the y-axis).
- Odd function: $f(-x) = -f(x)$ (graph is symmetric about the origin).
- Four ways to transform the graph of f:

$f(x) + c$	Shifts graph vertically $\lvert c\rvert$ units (upward if $c > 0$, downward if $c < 0$)
$f(x + c)$	Shifts graph horizontally $\lvert c\rvert$ units (to the right if $c < 0$, to the left if $c > 0$)
$kf(x)$	Scales graph vertically by factor k; if $k < 0$, graph is reflected across x-axis
$f(kx)$	Scales graph horizontally by factor k (compresses if $k > 1$); if $k < 0$, graph is reflected across y-axis

1.1 EXERCISES

Preliminary Questions

1. Give an example of numbers a and b such that $a < b$ and $|a| > |b|$.

2. Which numbers satisfy $|a| = a$? Which satisfy $|a| = -a$? What about $|-a| = a$?

3. Give an example of numbers a and b such that $|a + b| < |a| + |b|$.

4. Are there numbers a and b such that $|a + b| > |a| + |b|$?

5. What are the coordinates of the point lying at the intersection of the lines $x = 9$ and $y = -4$?

6. In which quadrant do the following points lie?

(a) $(1, 4)$ **(b)** $(-3, 2)$ **(c)** $(4, -3)$ **(d)** $(-4, -1)$

7. What is the radius of the circle with equation $(x - 7)^2 + (y - 8)^2 = 9$?

8. The equation $f(x) = 5$ has a solution if (choose one):

(a) 5 belongs to the domain of f.

(b) 5 belongs to the range of f.

9. What kind of symmetry does the graph have if $f(-x) = -f(x)$?

10. Is there a function that is both even and odd?

Exercises

1. Use a calculator to find a rational number r such that $|r - \pi^2| < 10^{-4}$.

2. Which of (a)–(f) are true for $a = -3$ and $b = 2$?

(a) $a < b$ **(b)** $|a| < |b|$ **(c)** $ab > 0$

(d) $3a < 3b$ **(e)** $-4a < -4b$ **(f)** $\frac{1}{a} < \frac{1}{b}$

In Exercises 3–8, express the interval in terms of an inequality involving absolute value.

3. $[-2, 2]$ **4.** $(-4, 4)$ **5.** $(0, 4)$

6. $[-4, 0]$ **7.** $[1, 5]$ **8.** $(-2, 8)$

In Exercises 9–12, write the inequality in the form $a < x < b$.

9. $|x| < 8$ **10.** $|x - 12| < 8$

11. $|2x + 1| < 5$ **12.** $|3x - 4| < 2$

In Exercises 13–18, express the set of numbers x satisfying the given condition as an interval.

13. $|x| < 4$ **14.** $|x| \le 9$

15. $|x - 4| < 2$ **16.** $|x + 7| < 2$

17. $|4x - 1| \le 8$ **18.** $|3x + 5| < 1$

In Exercises 19–22, describe the set as a union of finite or infinite intervals.

19. $\{x : |x - 4| > 2\}$

20. $\{x : |2x + 4| > 3\}$

21. $\{x : |x^2 - 1| > 2\}$

22. $\{x : |x^2 + 2x| > 2\}$

23. Match (a)–(f) with (i)–(vi).

(a) $a > 3$

(b) $|a - 5| < \frac{1}{3}$

(c) $\left|a - \frac{1}{3}\right| < 5$

(d) $|a| > 5$

(e) $|a - 4| < 3$

(f) $1 \le a \le 5$

(i) a lies to the right of 3.
(ii) a lies between 1 and 7.
(iii) The distance from a to 5 is less than $\frac{1}{3}$.
(iv) The distance from a to 3 is at most 2.
(v) a is less than 5 units from $\frac{1}{3}$.
(vi) a lies either to the left of -5 or to the right of 5.

24. Describe $\left\{x : \dfrac{x}{x+1} < 0\right\}$ as an interval. *Hint:* Consider the sign of x and $x + 1$ individually.

25. Describe $\{x : x^2 + 2x < 3\}$ as an interval. *Hint:* Plot $y = x^2 + 2x - 3$.

26. Describe the set of real numbers satisfying $|x - 3| = |x - 2| + 1$ as a half-infinite interval.

27. Show that if $a > b$, and $a, b \neq 0$, then $b^{-1} > a^{-1}$, provided that a and b have the same sign. What happens if $a > 0$ and $b < 0$?

28. Which x satisfies both $|x - 3| < 2$ and $|x - 5| < 1$?

29. Show that if $|a - 5| < \frac{1}{2}$ and $|b - 8| < \frac{1}{2}$, then $|(a + b) - 13| < 1$. *Hint:* Use the triangle inequality ($|a + b| \le |a| + |b|$).

30. Suppose that $|x - 4| \le 1$.
(a) What is the maximum possible value of $|x + 4|$?
(b) Show that $|x^2 - 16| \le 9$.

31. Suppose that $|a - 6| \le 2$ and $|b| \le 3$.
(a) What is the largest possible value of $|a + b|$?
(b) What is the smallest possible value of $|a + b|$?

32. Prove that $|x| - |y| \le |x - y|$. *Hint:* Apply the triangle inequality to y and $x - y$.

33. Express $r_1 = 0.\overline{27}$ as a fraction. *Hint:* $100r_1 - r_1$ is an integer. Then express $r_2 = 0.2666\ldots$ as a fraction.

34. Represent $1/7$ and $4/27$ as repeating decimals.

35. The text states: *If the decimal expansions of numbers a and b agree to k places, then* $|a - b| \le 10^{-k}$. Show that the converse is false: For all k there are numbers a and b whose decimal expansions *do not agree at all* but $|a - b| \le 10^{-k}$.

36. Plot each pair of points and compute the distance between them:

(a) (1, 4) and (3, 2)

(b) (2, 1) and (2, 4)

(c) (0, 0) and (−2, 3)

(d) (−3, −3) and (−2, 3)

37. Find the equation of the circle with center (2, 4):
(a) with radius $r = 3$.
(b) that passes through $(1, -1)$.

38. Find all points in the xy-plane with integer coordinates located at a distance 5 from the origin. Then find all points with integer coordinates located at a distance 5 from (2, 3).

39. Determine the domain and range of the function

$$f : \{r, s, t, u\} \to \{A, B, C, D, E\}$$

defined by $f(r) = A$, $f(s) = B$, $f(t) = B$, $f(u) = E$.

40. Give an example of a function whose domain D has three elements and whose range R has two elements. Does a function exist whose domain D has two elements and whose range R has three elements?

In Exercises 41–48, find the domain and range of the function.

41. $f(x) = -x$

42. $g(t) = t^4$

43. $f(x) = x^3$

44. $g(t) = \sqrt{2 - t}$

45. $f(x) = |x|$

46. $h(s) = \dfrac{1}{s}$

47. $f(x) = \dfrac{1}{x^2}$

48. $g(t) = \cos \dfrac{1}{t}$

In Exercises 49–52, determine where f is increasing.

49. $f(x) = |x + 1|$

50. $f(x) = x^3$

51. $f(x) = x^4$

52. $f(x) = \dfrac{1}{x^4 + x^2 + 1}$

In Exercises 53–58, find the zeros of f and sketch its graph by plotting points. Use symmetry and increase/decrease information where appropriate.

53. $f(x) = x^2 - 4$

54. $f(x) = 2x^2 - 4$

55. $f(x) = x^3 - 4x$

56. $f(x) = x^3$

57. $f(x) = 2 - x^3$

58. $f(x) = \dfrac{1}{(x - 1)^2 + 1}$

59. Which of the curves in Figure 26 is the graph of a function?

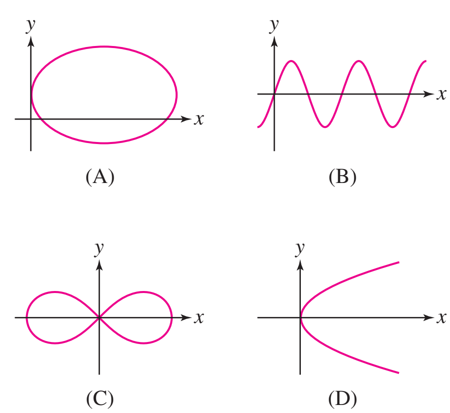


FIGURE 26

60. Determine whether the function is even, odd, or neither.

(a) $f(x) = x^5$

(b) $g(t) = t^3 - t^2$

(c) $F(t) = \dfrac{1}{t^4 + t^2}$